Improving the accuracy of a discrete repetitive controller applied to an active power filter when the mains frequency is unknown

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***Abstract*—Repetitive controllers are used to track periodic references or to reject periodic disturbances, exactly. When implementing this type of controllers digitally, the sampling period has to be chosen so that there is an integer number of samples within a period of the signal to be tracked or rejected. Otherwise, the controller performance deteriorates dramatically. This requirement is not always easy to fulfil when the signal period is not known exactly at the design stage or when it changes during operation. This paper presents a simple addition for a first-order discrete repetitive controller to be used in applications in which the period of the reference signal or disturbance is not an exact multiple of the sampling period. The algorithm is proposed to tackle the problem of small errors in the mains frequency estimation for an active power filter to be used in an electrical-energy distribution system. The paper illustrates how the results of a previously-tested active power filter are improved with the addition proposed.**

I. Introduction

Repetitive controllers (RCs) have been proposed in the literature to track periodic references or to reject periodic disturbances with zero error in steady state [1]. Ideal steady-state performance is guaranteed if the signal period (T in the rest of the paper) is known exactly and the sampling period (t_s in the rest of the paper) can be chosen so that one cycle of the periodic signal consists of an integer number of samples [2]. Precision deteriorates if only a small error in the signal frequency is present [3] or an integer period ratio (T/t_s) cannot be achieved. In fact, the latter case is seen by the controller as if there was an error in the a-priori estimated signal period. Robust higher-order repetitive controllers (HORC) have been proposed in [4] to tackle this problem with the disadvantages that it is necessary to store more than one cycle of the periodic signals involved, they show slower response and closed-loop stability is more difficult to guaranty than in the case of a simpler RC.

This paper proposes a simple addition that greatly improves the performance of a first-order RC (FORC) when the signal period is known but it is not an integer multiple of the sampling period. This approach has been tailored to be implemented in shunt active power filters (SAPF) where a RC has proved to be very useful if the ideal conditions are exactly met [3]. However, ideal conditions may not always be true in weak grids or micro-grids working with distributed generation, for example, where relatively-large frequency excursions around 50 Hz (or 60 Hz) are likely. Nevertheless, the mains frequency can be easily estimated on-line (see [5] and [6]) and the number of samples per cycle can be rounded to the nearest integer number towards zero. The proposed algorithm will then improve the controller robustness if the period ratio is not an integer number.

This paper is structured as follows. Section II describes a shunt active power filter application, briefly. Section III describes the fundamentals of repetitive controllers and explains the implementation proposal. Section IV illustrates the effects of the proposed addition. Section V illustrates the performance improvement of the proposed algorithm when compared to a typical first-order repetitive controller and a higher-order repetitive controller. Section VI shows the results of a simplified laboratory prototype. Finally, Section VII summarizes the main contributions of the paper.

II. Preliminaries: Overview of shunt active power filter control

A SAPF to compensate non-linear load currents is depicted in Fig. 1. The currents drawn by the non-linear load and supplied by the filter are i_L and i_{AF}, respectively. The case discussed here is that in which the former is periodic showing a fundamental component of the mains frequency together with an unknown harmonic contents, in steady state. The primary function of the SAPF is to supply the current harmonics drawn by the load so that the mains current i_S is an ideal sinusoid. Therefore, the load current is measured and its harmonic components extracted (i_{Lh}) to be a part of the SAPF reference current ($i^r_{AFh} = i_{Lh}$). In addition, the SAPF may be required to compensate the unbalance of the load current and to supply/consume a certain amount of reactive power. These tasks will determine an extra term of the mains frequency to be added to the filter-current reference (i^r_{AF1}). In order to cancel a given harmonic (h), the SAPF must follow its reference with zero phase and unity gain. The current injected by the SAPF

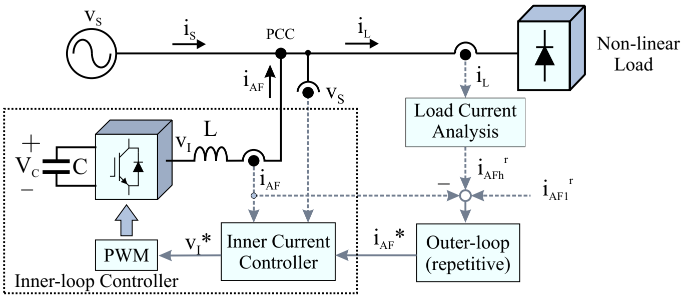

Fig. 1. Typical set-up for a shunt active power filter (SAPF)

will always be beneficial if the phase lag with respect to its reference is less than 60 degrees.

The SAPF is based on a PWM voltage-source inverter with a constant D.C. input voltage V_c and it is connected to the "point of common coupling" (PCC) through, at least, an inductance L with an internal resistance R. This inductance will prevent connecting two voltage sources (namely, v_S and v_I) in parallel and will help to filter the inverter-switching frequency.

In any case, the SAPF will be challenged with the accurate tracking of a current reference that contains harmonics of the mains frequency. Since a voltage-source inverter is used, closed-loop control of the current through L (i_{AF}, in Fig. 1) is required. Although this could be done with a single-layer controller with high sampling frequency such as in [7] or [8], the work in [3] proposes a two-layer linear control strategy with relatively-low sampling frequency: an inner current-control loop and an outer control loop. The latter produces the reference values for the former. The so-called repetitive controller is very appropriate for the outer loop (see Fig, 1).

III. Fundamentals of Repetitive Controllers and a Proposed Addition

The core of a RC is a transfer function (using Laplace variable s) such as:

$$C_o(s) = \frac{e^{-Ts}}{1 - e^{-Ts}} \tag{1}$$

If the sampling period of the digital controller to be implemented (t_s) is such as:

$$T = Nt_s + l_e t_s \tag{2}$$

with N a positive integer number and

$$l_e \in (0, 1) \tag{3}$$

The controller in Eq. (1) can be written as:

$$C_o(s) = \frac{e^{-Nt_s s} e^{-l_e t_s s}}{1 - e^{-Nt_s s} e^{-l_e t_s s}} \tag{4}$$

which shows (as required) large magnitude at all frequencies of the form

$$\omega_h = h \frac{2\pi}{Nt_s + l_e t_s} rad/s, \; (h = 0, 1, 2, \cdots) \tag{5}$$

which are included in all signals of period T.

For a discrete-time implementation, the core of the RC can be derived from Eq. (1), with the following steps:

$$\begin{aligned}
&(a) \qquad e^{-Nt_s s} \text{ changes to } z^{-N} \\
&(b) \qquad e^{-l_e t_s s} \approx \frac{1 - \frac{t_s l_e}{2} s}{1 + \frac{t_s l_e}{2} s} \;\; (\text{Pade's approx.}) \\
&\qquad\qquad \text{changes to } (\text{Tustin's approx.}) \\
&\qquad\qquad L(z) = \frac{(1 - l_e) + (1 + l_e)\, z^{-1}}{(1 + l_e) + (1 - l_e)\, z^{-1}}
\end{aligned} \tag{6}$$

Taking into account steps (a) and (b) described above, the proposal for a digital repetitive controller is depicted in Fig. 2, where $P(z)$ is the discrete-time model of the plant, K_x is a user-defined gain, $G_x(z)$ is a user-defined discrete-time transfer function which, mainly, compensates the plant delay to make closed-loop stability possible and $Q(z)$ is a low-pass filter to disconnect the RC at high frequencies where $G_x(z)$ is not effective (see [2], for details). Regarding the example in Fig. 1, the plant $P(z)$ is the SAPF with the inner current loop working.

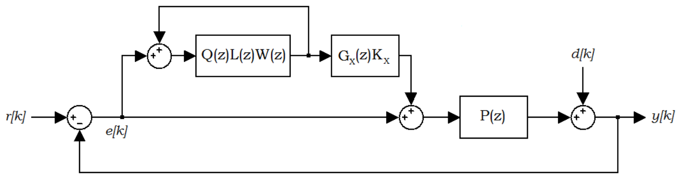

Fig. 2. Digital repetitive controller

Fig. 2 will be a first-order repetitive controller (FORC) if,

$$W(z) = z^{-N} \;\; [FORC] \tag{7}$$

and, it will be a higher-order repetitive controller (HORC) if,

$$W(z) = \sum_{i=1}^{m} W_i z^{-iN} \;\; [HORC] \tag{8}$$

The weights W_i for $W(z)$ in Eq. (8) can be chosen to improve robustness against period errors as suggested by [4]. $L(z)$ in Figure 2 is, traditionally, made equal to 1 but it is proposed to be like in Eq. (6), in this paper.

The Z-transform of the error signal (E(z)) in Fig. 2 can be written as follows for a given reference input ($R(z)$), if disturbance ($D(z)$) is neglected. Similar analysis can be done for a periodic disturbance:

$$E(z) = \left[\frac{1}{1 + P(z)}\right] \left(\frac{1 - Q(z)L(z)W(z)}{1 - Q(z)L(z)W(z)\,[1 - K_x G_x(z) G_p(z)]}\right) R(z) \tag{9}$$

with

$$G_p(z) = \frac{P(z)}{1 + P(z)} \tag{10}$$

Therefore, the closed-loop system in Fig. 2 is stable if [2]:

$(a) \qquad G_p(z)$ is stable and

$$(b) \qquad max\,|W(z)Q(z)L(z)|\,|1 - K_x G_x(z) G_p(z)| < 1$$
$$\text{with } z = e^{j\omega t_s} \text{ for all } |\omega| < \frac{\pi}{t_s} \tag{11}$$

To start with, one may have to add an extra compensator in series with the plant to be controlled in order to ensure the stability of $G_p(z)$. This compensator should be include as part of $P(z)$ in the analysis to follow. Finally, stability is easily guaranteed because $G_x(z)$ is typically made:

$$G_x(z) = G_p^{-1}(z) = \left(\frac{P(z)}{1 + P(z)}\right)^{-1} \tag{12}$$

Besides, since $\left|L(e^{j\omega t_s})\right| = 1$ in Eq. (11), $L(z)$ does not affect stability. $L(z)$ has been derived from [9] where, unfortunately, stability was not maintained. Finally, it is worth pointing out that the weighting function may complicate stability since it is possible to have $|W(z)| > 1$ within the frequency range of interest.

IV. The effect of the $L(z)$ addition in the repetitive controller

If the system is made closed-loop stable, Eq. (9) shows that the steady state error is zero for all frequencies included in the periodic reference (fundamental $\omega_1 = (2\pi/T)\,rad/s$ and its harmonics $\omega_h = h\omega_1$) if:

$$|M(z)| = |1 - Q(z)L(z)W(z)| = 0 \text{ with } z = e^{jt_s\omega_h} \tag{13}$$

Since the core of the repetitive controller is

$$R_c(z) = \frac{Q(z)L(z)W(z)}{1 - Q(z)L(z)W(z)} \tag{14}$$

it will show infinite gain for those frequencies.

Filter $Q(z)$ has no implication in Eq. (13) because, within its bandwidth, $|Q(z)| = 1$ and the phase of $Q(z)$ can be made equal to zero if a FIR filter is used (see [2], for details). If N samples do not span exactly a period of the reference to follow, Eq. (13) is not satisfied at, exactly, the fundamental and harmonic frequencies considered. However, it can be shown that, in this case, the error is reduced if $L(z)$ as in Eq. (6) is used. The precision may be improved further if a higher-degree Pade's approximation is used in Eq. (6). In order to illustrate this point, the magnitude of the frequency response of $R_c(z)$ has been plotted for the 20th harmonic of the reference signal in Fig. 3, using $L(z)$. The design fundamental frequency has been made equal to 50 Hz, the sampling frequency has been made equal to 5 kHz while the actual base frequency of the periodic signal to be followed has been considered to be between 50 Hz and 50 Hz+1% with frequency increments of 0.25%. The frequencies on the x-axis of Fig. 3 can be calculated as $20 \times (1 + 0.01 \times n) \times 50\ Hz$. The ideal case is when the design frequency and the actual frequency to be followed coincide ($l_e = 0$ and $N = 100$): the 20th harmonic will be in $n = 0$ and the repetitive controller peak will be there too. As the actual frequency moves to $n = 0.25$, etc. $L(z)$ forces the peak to move close to the required frequency of the 20th harmonic. If $L(z)$ is not used, the controller peak remains in $n = 0$ while the harmonic frequency moves. When $n = 1$, one could again achieve ideal performance without $L(z)$ if N is changed to $N = 99$.

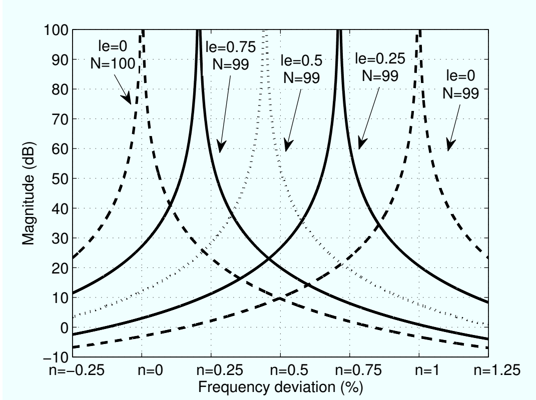

Fig. 3. Bode-plot magnitude of repetitive control block, $R_c(z)$, with $L(z)$

V. Precision results for a three-phase SAPF

A SAPF with a traditional FORC ($L(z) = 1$) was evaluated experimentally in [3] using the structure of Fig. 1. The repetitive controller was designed with the following closed-loop discrete transfer function for the inner current controller (sampling frequency equal to $5kHz$):

$$\frac{i^f_{AFd}(z)}{i^*_{AFd}(z)} = \frac{0.05357}{z^4 - 2.515z^3 + 2.549z^2 - 1.199z + 0.2187} \tag{15}$$

where superscript f in Eq. (15) means "filtered" since the actual currents were measured through an anti-aliasing filter. The asterisk marks the reference value for the inner current loop. Finally, subscript d stands for "$d - axis$". A similar transfer function can be obtained for the "$q - axis$". Notice that $d-$ and $q - axis$ currents can be obtained from the three-phase currents using Park's Transformation [10].

Filter $Q(z)$ was made equal to:

$$Q(z) = \frac{1}{4}z + \frac{1}{2} + \frac{1}{4}z^{-1} \tag{16}$$

The repetitive controller was shown to work adequately in [3] if one cycle of the mains period contained an integer number of samples.

The error signal in Eq. (9) has been evaluated in steady state for the above-mentioned SAPF with three different proposals of repetitive controller with a sampling frequency equal to $5kHz$. The evaluation was carried out using a expected mains frequency (f_{10}) equal to $50Hz$ for design purposes, while the actual mains frequency ($f_1 = 1/T$) was changed so that,

$$0 \leq 100\frac{f_1 - f_{10}}{f_{10}} \leq 1 \tag{17}$$

Since the performance of the repetitive controller is considered mainly within its bandwidth, the Q filter has been made ideal ($Q(z) = 1$) for the calculations shown bellow. The error signal has been evaluated in steady state at the fundamental and harmonic frequencies. The three alternatives for the repetitive controller can be summarized as follows:

1) A typical FORC with a fixed $N = 100$. This is as in the case reported in [3]. $|E(z)|$ in steady state is presented in Fig. 4.
2) A HORC (third order, $m = 3$) as proposed in [4] where the weights in Eq. (8) were calculated for period robustness and N was changed to the nearest-possible integer number after dividing the sampling frequency ($1/t_s$) by the actual mains frequency considered (f_1). $|E(z)|$ in steady state and the weight values W_i are presented in Fig. 5.
3) A FORC with the addition of $L(z)$ as proposed in Eq. (6) and changing N according to Eq. (2). $|E(z)|$ in steady state is presented in Fig. 6.

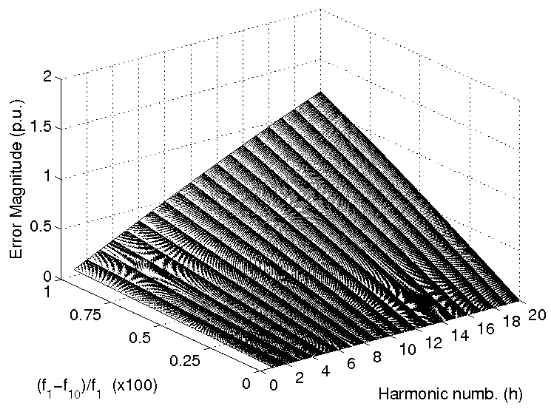

Fig. 4. Steady-state error amplitude when reference amplitude for the repetitive controller is equal to 1 for each frequency component (h). FORC with N=100.

Furthermore, a detailed numerical comparison of the steady-state error amplitude for a typical FORC and the proposed algorithm is presented in Table I. The former is characterized by $l_e = 0$ in the last column of Table I while the latter is characterized by $l_e > 0$. In the typical FORC the best-possible number of samples (N) per cycle of the fundamental frequency (T) has been used. This adaptation of N already improves the performance with respect to the one presented in Fig. 4.

The results obtained show that a plain FORC has a poor performance when the actual reference period is not exactly an integer number of samples in the digital controller. The period-robust HORC proposal in [4] greatly improves the controller performance if N is chosen as the nearest-possible integer. However, the simple proposal in this paper is superior using a FORC requiring the minimum memory size.

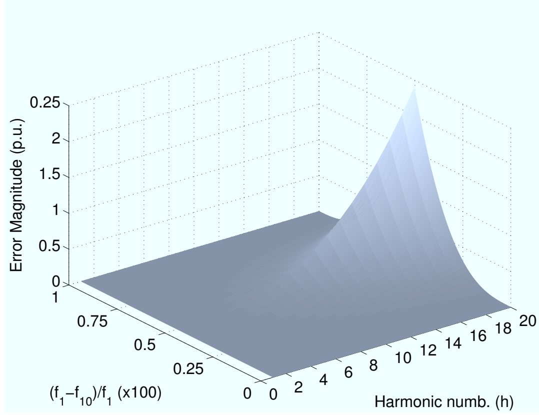

Fig. 5. Steady-state error amplitude when reference amplitude for the repetitive controller is equal to 1 for each frequency component (h). HORC (3rd order). N is the nearest-possible integer. $W_1 = 3$, $W_2 = -3$ and $W_3 = 1$

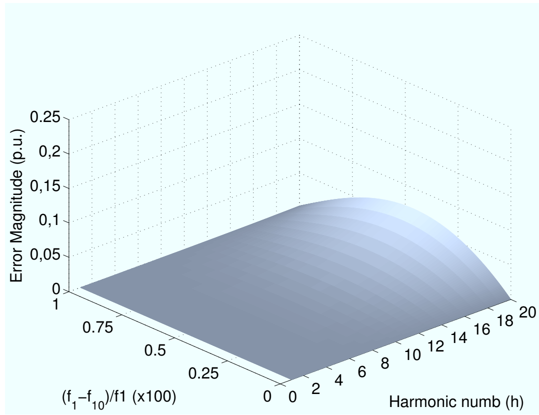

Fig. 6. Steady-state error amplitude when reference amplitude for the repetitive controller is equal to 1 for each frequency component (h). FORC with the proposed addition.

VI. Testing a simple single-phase SAPF

The proposed algorithm has also been tested experimentally with a simplified version of the system depicted in Fig. 1. The PWM inverter in the filter has been simulated with an audio amplifier TDA7294 in the prototype and a 25th-order FIR filter has been used in $Q(z)$. The mains frequency has been estimated using a Kalman Filter [11] to update N and calculate $L(z)$ in the repetitive controller, if required. The mains frequency in the laboratory was measured and it was confirmed to be 50 Hz. In order to test the effect of using $L(z)$, the controller sampling frequency was made equal to 4.525 kHz so that $N = 90$ and $l_e = 0.5$. The controller has been run in a PC using SIMULINK-MATLAB XPC-target.

Figure 7 shows the SAPF performance when the repetitive controller is switched "on" ("pluged in") at $40\ ms$ after the other control systems are working. The amplifier output voltage V_I (top), the amplifier current I_{AF} and the mains current I_S are plotted. The detailed harmonic contents of the load and mains currents in steady state, with the repetitive

f1/f10	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	N	Ie
1.001	0.0000	0.0000	0.0001	0.0002	0.0004	0.0008	0.0012	0.0018	0.0026	0.0035	99	0.9001
1.001	0.0063	0.0126	0.0188	0.0251	0.0314	0.0377	0.0440	0.0503	0.0565	0.0628	100	0
1.005	0.0000	0.0001	0.0002	0.0005	0.0010	0.0017	0.0027	0.0041	0.0059	0.0081	99	0.5025
1.005	0.0314	0.0628	0.0942	0.1256	0.1569	0.1882	0.2195	0.2507	0.2818	0.3129	100	0
1.008	0.0000	0.0000	0.0001	0.0003	0.0005	0.0009	0.0015	0.0022	0.0031	0.0043	99	0.2063
1.008	0.0131	0.0261	0.0392	0.0523	0.0653	0.0784	0.0915	0.1045	0.1176	0.1306	99	0
1.010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	99	0.0099
1.010	0.0006	0.0013	0.0019	0.0025	0.0031	0.0038	0.0044	0.0050	0.0057	0.0063	99	0
1.020	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0003	0.0005	0.0006	0.0009	98	0.0392
1.020	0.0025	0.0050	0.0075	0.0101	0.0126	0.0151	0.0176	0.0201	0.0226	0.0251	98	0

TABLE I
DETAILED COMPARISON BETWEEN A TRADITIONAL FORC AND THE PROPOSED ALGORITHM.

controller, are plotted in Fig. 8. The repetitive controller shows good performance even if the ratio between mains period and sampling period is not an integer number.

VII. CONCLUSIONS

Repetitive controllers require some sort of tuning when the reference period is not known exactly at the design stage. In discrete-time applications, this will surely lead to a number of samples which do not cover a full period of the reference signal, exactly. This paper shows that a simple filter can be added to the traditional RC to tackle this problem. In addition, this filter does not compromise stability of the closed-loop system.

The contribution of the proposed algorithm has been shown using the error transfer function of an active power filter already tested experimentally. An experiment has also been run in order to investigate the results further.

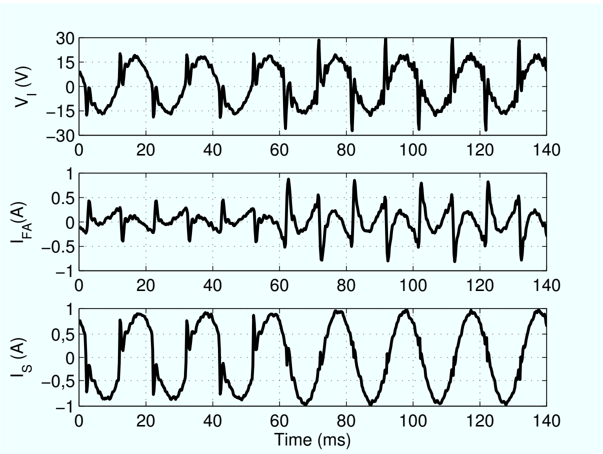

Fig. 7. Transient and steady-state performance of a repetitive controller in a single-phase SAPF

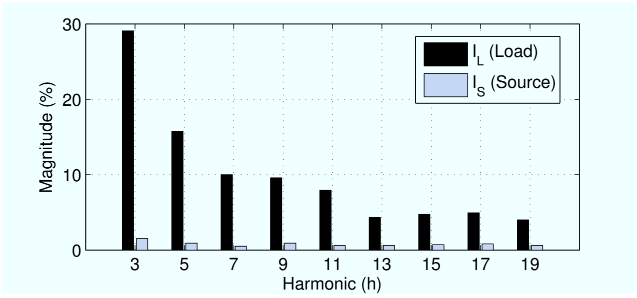

Fig. 8. Steady-state load and mains currents with a SAPF

ACKNOWLEDGMENTS

The authors thank the support of the Spanish Government research grant ENE2006-13705.

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